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Nr. 199

Arbeitspapiere der
Volkswirtschaftlichen Fachgebiete der TU Darmstadt

ISSN: 1438-2733

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October 13, 2010

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Abstract

In this paper we specialize the Ngai-Pissarides model of growth and structural change [American Economic Review 97 (2007), 429-443] to the case of three sectors, representing the primary (agriculture, mining), secondary (construction, manufacturing) and tertiary (services) sectors. On that basis we explore the dynamic properties of the model along the transition path to the steady-state equilibrium by numerical methods. Our explorations show that the model misses several stylized facts of structural change among these sectors. We propose several extensions of the model to align the model more closely with the facts.

JEL classification: L16, O14, O41

keywords: economic growth, structural change, transition path

1 Introduction

Economic development and aggregate economic growth are inevitably associated with a changing sectoral composition of the economy. For the period since the onset of the industrial revolution we can observe a very specific pattern of structural change among the three main sectors of the private economy. According to this pattern the primary sector (agriculture, mining) is dominating the economy before the onset of industrialization, then the secondary sector (manufacturing, construction) begins to gain in importance and grows until the tertiary sector (services) starts to take off. This leads to the characteristic declining share of the primary sector, the rising share of the tertiary sector and the hump-shaped share of the secondary sector which can be observed for many countries since the 19th century.

Formal economic growth theory pays little attention to this phenomenon, however. Even unified growth theory, which is particularly dedicated to the explanation of the long-run since the Middle Ages, is more focused on demographic change and human capital formation than on structural change (see Galor, 2005 for a survey of this literature). Just a small literature is concerned with integrating uneven development of sectors and therefore structural change into growth models, namely Echevarria (1997), Kongsamut et al. (2001), Meckl (2002), Ngai and Pissarides (2007), and Foellmi and Zweimüller (2008). Recently, these economists pay more attention to the puzzle whether the Kuznets facts (systematic change of the sector shares in employment and value added) can coexist with the Kaldor facts (a constant interest rate, constant growth rate, constant capital-output ratio, and a constant labor income and capital income share from GDP).

In this paper one of these models is taken, specialized to three sectors and its ability to reproduce the typical pattern of structural change along the transition path to the steady-state equilibrium using numerical methods for solving the associated differential equations is explored. The model of Ngai and Pissarides (2007), henceforth NP, is chosen as the basis for the investigation since this model is one of the very few that can replicate a hump-shaped sectoral share. In contrast to the analysis of NP who investigate structural change as a steady-state phenomenon we are interested in the pattern of structural change along the transition path to this steady state.

The structure of the paper is as follows: Following this introduction, section 2 contains an account of the empirical evidence on the characteristic three-sector development together with a very brief account of growth models that contain structural change. These papers are discussed in much more detail in the survey paper of Krüger (2008). In section 3 the key equations of the original NP model, specialized to our objective, are presented. Section 4 gives a discussion of the approach used for the numerical solution of the saddle-path stable steady state. This section also justifies the parameter values used for the calibration exercise which is presented in the next section 5. The following sections deal with modifications of the original model: In section 6 the modeling of sectoral productivity growth differences is modified in order to be closer to the empirical findings for the long-run development of sectoral productivity growth. The next section 7 introduces non-homothetic preferences to be allowed to also explore the influence of the demand side in long-run sectoral development. Due to criticism of neoclassical growth theory, section 8 is an attempt to endogenize technological change and section 9 adds human capital accumulation. In all cases, the ability of the modifications to improve the models capability to explain the empirical facts is thoroughly evaluated. Finally, the main results are summarized in the concluding section 10.

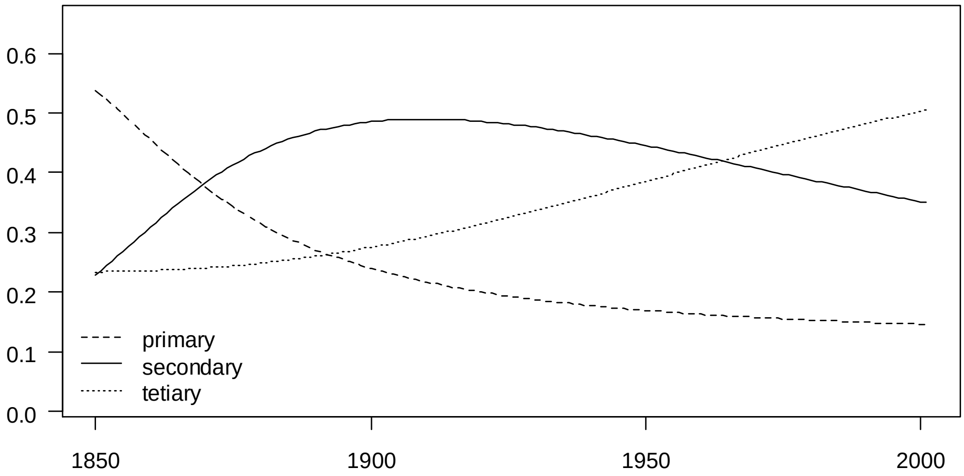
2 Empirical evidence

In most developed countries long-run economic growth was associated with a specific pattern of structural change among the three main sectors of the private economy. Before the onset of industrialization most economic activity took place in the primary sector comprising agricultural and natural resource extracting activities (mining). Production of goods was limited to handicraft and services played even less a role. With the onset of industrialization the accumulation of capital and the production of goods on a larger scale became increasingly important. This triggered labor flows from the primary to the secondary (manufacturing) sector. After World War II more and more economic activity moved from the secondary to the tertiary sector (services) and this process of tertiarization appears to be going on today.

Kuznets (1966) documents this pattern for 13 OECD countries and the USSR between 1800 and 1960 and Maddison (1980) extends this work to 16 OECD countries from 1870 to 1987. Both found the same general pattern of sectoral development for all investigated countries. Furthermore, Feinstein (1999) provides a comprehensive empirical account of the three-sector development in 25 countries during the 20th century, confirming the typical pattern in all countries considered. Scattered evidence from emerging economies suggests that structural change proceeds rather similar there, although on a compressed time scale. The typical development is accordingly to these findings very similar for all economies even if the curvature is not the same in all countries (Kaelble, 1997).

Figure 1 shows the stylized development of the three sector shares with a monotonically decreasing share of the primary sector, a monotonically rising share of the tertiary sector and with the share of the secondary sector following a hump-shaped pattern.

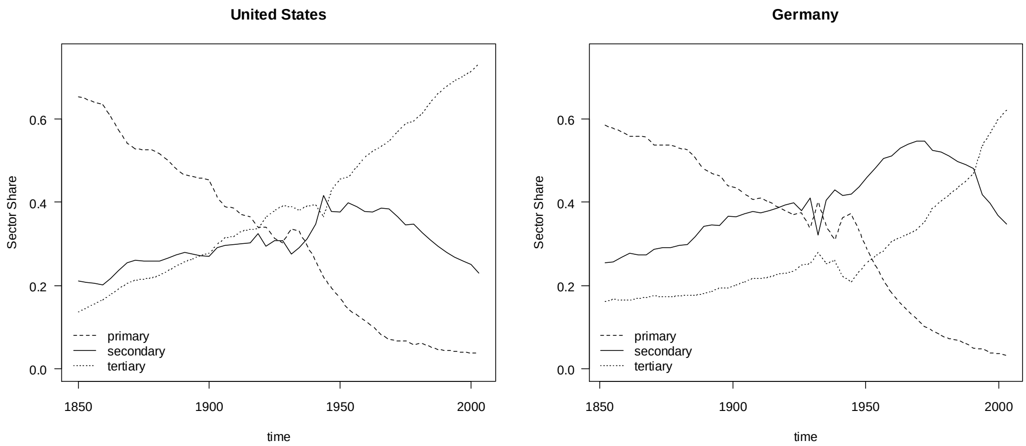
Figure 1: Stylized sectoral development



Source: Own illustration.

This characteristic pattern is also clearly visible in historical data, although of course scarred by crises, wars and exceptional events. Figure 2 depicts the development of the employment shares of the U.S. and the German economy during the period 1850-2003.[1] Interestingly, the German economy seems to lag behind the U.S. for several decades concerning the decline of the secondary and the rise of the tertiary sector.

Figure 2: Sectoral employment development of the U.S. and Germany since 1850



Source: Data taken from various sources, see footnote 1.

[1]The data measure employment shares of the private economy and are taken from Carter et al. (2006), van Ark (1996), and O'Mahony and van Ark (2003) for the U.S. economy as well as from Hoffmann (1965), Wagenführ (1963), van Ark (1996), and O'Mahony and van Ark (2003) for the German economy. Note that data are available only at 10-year intervals until 1900 for the U.S. economy and data gaps appear for some years between 1850 and 1878 for the German economy. Further data are missing from 1914-1924 and 1945-1949 for Germany. These data gaps have been linearly interpolated in the graphs, letting the curves appear somewhat smoother than they actually are.

In the economic literature different causes for structural change in general and this particular pattern are discussed. A first technological explanation is focused on the supply side and identifies different rates of productivity growth between the sectors as the primary source of structural change. This approach can be traced back to the work of Fisher (1945) and Baumol (1967) and is also called productivity hypothesis. A second explanation highlights demand-side causes for structural change. This approach is also called utility-based explanation and can be traced back to Fisher (1939, 1952) and Fourastié (1949, 1969). Here mainly non-homothetic preferences or a hierarchy of needs are the main drivers for structural change. In this case structural change can take place even with identical productivity growth rates among the sectors.

Up to now, there is no clear evidence whether technological progress or changes in the preferences is the driving force of structural change. Baumol et al. (1985) and more recently, Nordhaus (2008) provide empirical evidence favoring the technological explanation on a two digit industry level. Kravis et al. (1983) confirm these findings for manufacturing and services. By contrast, Dietrich and Krüger (2010) found empirical evidence more in favor of the demand-driven explanation of the three sectoral patterns for the German economy. Additionally, authors like Curtis and Murthy (1998), Rowthorn and Ramaswamy (1999), and Möller (2001) have shown that the income elasticity is greater than unity for most service branches as well as for aggregate services and below unity for manufacturing branches as well as the sector as aggregate which is also a hint favoring the utility-based explanation. In Krüger (2008) a broad survey on these theoretical explanations and related empirical investigations of structural change overall and especially for the case of the three sector hypothesis is given.

Within formal growth theory the papers of Echevarria (1997), Kongsamut et al. (2001), Meckl (2002), and Foellmi and Zweimüller (2008) feature both demand- or supply-side causes of structural change among the three or more sectors in which the economy is divided. These papers are also surveyed in Krüger (2008).

A particularly interesting paper in this respect is Ngai and Pissarides (2007), henceforth NP, proposing a formal general equilibrium growth model in which differential exogenous rates of productivity growth are stressed as the primary cause of structural change. The demand side is governed by homothetic preferences and thus no cause for structural change. The model, however, has the property that employment shares of some sectors may show a hump-shaped development over time. This feature makes it a promising starting point for our numerical explorations in the present paper.

3 The original Ngai/Pissarides model

The NP model is a neoclassical growth model that describes an economy with an arbitrary number of m sectors, where $m-1$ sectors produce only consumption goods and one sector produces a final consumption good and the economies capital stock. Here, we investigate the special case of three sectors and interpret these sectors as the primary (agriculture), secondary (manufacturing), and tertiary (services) sector of the economy, respectively. Capital accumulation takes place in the secondary sector, while the primary and the tertiary sector produce only consumption goods.

As usual the objective is to maximize the present value of utility derived from the consumption of the goods of the three sectors

$$U = \int_0^\infty e^{-\rho t} u\left(c_a, c_m, c_s\right) \tag{1}$$

where $\rho > 0$ is the rate of time preference, c_i denote per capita consumption levels, and the utility function $u\left(.\right)$ is concave and satisfies the Inada conditions such as

$$u\left(c_a, c_m, c_s\right) = \frac{\varphi\left(c_a, c_m, c_s\right)^{1-\theta} - 1}{1-\theta} \tag{2}$$

The sectoral consumption are aggregated by the familiar CES aggregator function

$$\varphi\left(c_a, c_m, c_s\right) = \left(\omega_a c_a^{(\varepsilon-1)/\varepsilon} + \omega_m c_m^{(\varepsilon-1)/\varepsilon} + \omega_s c_s^{(\varepsilon-1)/\varepsilon}\right)^{\varepsilon/(\varepsilon-1)} \tag{3}$$

with $\varepsilon > 0, \omega_a, \omega_m, \omega_s > 0$ and $\omega_a + \omega_m + \omega_s = 1$.[2]

On the production side capital and labor are used to produce the consumption goods of the primary and the tertiary sector via a Cobb-Douglas production function under constant returns to scale

$$c_i = y_i = A_i \left(n_i k_i\right)^\alpha n_i^{\alpha-1} = A_i n_i k_i^\alpha \text{ for } i = a, s \tag{4}$$

with $0 < \alpha < 1$ and $y_i, A_i, n_i,$ and k_i denoting the output, the productivity level measured as total factor productivity (TFP), the employment share of the sector as labor input, and the capital-labor ratio, respectively. The labor force is assumed to grow at the constant exogenous rate ν. In the secondary sector the produced output is used for consumption and for capital accumulation according to

$$\dot{k} = y_m - c_m - \left(\delta + \nu\right) k = A_m n_m k_m^\alpha - c_m - \left(\delta + \nu\right) k \tag{5}$$

[2]These functions become logarithmic for $\theta = 1$ and $\varepsilon = 1$.

with $0 < \delta < 1$ as the depreciation rate. k denotes the aggregate capital-labor ratio and is the weighted sum of sector-specific capital-labor ratios

$$n_a k_a + n_m k_m + n_s k_s = k \tag{6}$$

The static equilibrium conditions on the production side imply that the capital-labor ratio is the same in the three sectors so that this condition is trivially fulfilled since the employment shares add up to unity

$$n_a + n_m + n_s = 1 \tag{7}$$

The production functions are assumed to be identical across all sectors, except for their sector specific productivity growth rates

$$\frac{\dot{A}_i}{A_i} = \gamma_i \text{ for } i = a, m, s \tag{8}$$

NP derive the static and dynamic efficiency conditions for this model which imply

$$\frac{p_i}{p_m} = \frac{\partial u / \partial c_i}{\partial u / \partial c_m} = \frac{A_m}{A_i} \text{ for } i = a, m \tag{9}$$

where p_i denotes the price of good i. These conditions are used to define a new variable x_i as the ratio of consumption expenditures for good i to consumption expenditures for the manufacturing good produced in the secondary sector by

$$\frac{p_i c_i}{p_m c_m} = \left(\frac{\omega_i}{\omega_m}\right)^{\varepsilon} \left(\frac{A_m}{A_i}\right)^{1-\varepsilon}. \tag{10}$$

Aggregate consumption expenditure and output per capita are then defined in terms of manufacturing as the Numéraire by

$$c = \frac{p_a}{p_m} c_a + c_m + \frac{p_s}{p_m} c_s \tag{11}$$

and

$$y = \frac{p_a}{p_m} y_a + c_m + \frac{p_s}{p_m} y_s. \tag{12}$$

Using the static efficiency conditions it can be derived that

$$c = c_m X \tag{13}$$

where $X \equiv x_a + x_m + x_s$ and

$$y = A_m k^{\alpha}. \tag{14}$$

NP define structural change is defined as a state in which employment shares are changing over time in at least some sectors. The shares of the labor force that are engaged in non-manufacturing sectors are given by

$$n_i = \frac{x_i}{X}\left(\frac{c}{y}\right) \text{ for } i = a, s \tag{15}$$

and the share for the manufacturing sector is given by

$$n_m = \frac{x_m}{X}\left(\frac{c}{y}\right) + \left(1 - \frac{c}{y}\right). \tag{16}$$

The latter equation reflects the special position of the manufacturing sector where, in contrast to the other sectors, employment is not only needed for producing consumption goods, but also for capital accumulation. Note that the part in the second bracket of Eq. (16) is equal to the savings rate. Eq. (15) and (16) drive the structural change results of this model.

Aggregate dynamics are governed by the following two differential equations

$$\dot{k} = A_m k^{\alpha} - c - (\delta + \nu) k \tag{17}$$

$$\dot{c} = c\left[(\theta - 1)(\gamma_m - \bar{\gamma}) + \alpha A_m k^{\alpha-1} - (\delta + \rho + \nu)\right]/\theta \tag{18}$$

where $\bar{\gamma} = (x_a\gamma_a + x_m\gamma_m + x_s\gamma_s)/X$ denotes the weighted average productivity growth rate.

Aggregate balanced growth is defined in a way that aggregate output, consumption, and capital grow at the same rate. For the derivation of the steady state, aggregate consumption, and the capital labor ratio are defined in terms of efficiency units $c_e = c/A_m^{1/(1-\alpha)}$ and $k_e = k/A_m^{1/(1-\alpha)}$. The differential equations become

$$\dot{k_e} = k_e^{\alpha} - c_e - (g_m + \delta + \nu) k_e \tag{19}$$

$$\dot{c_e} = c_e\left[\alpha k_e^{\alpha-1} + \psi - (\delta + \rho + \nu)\right]/\theta - g_m \tag{20}$$

where $\psi = (\theta - 1)(\gamma_m - \bar{\gamma})$ and $g_m = \gamma_m/(1 - \alpha)$.

The necessary condition for a balanced growth path of the model is ψ being constant. This can either be guaranteed by imposing that the utility function is logarithmic (the case $\theta = 1$) or by the restriction that the productivity growth rates are equal across all sectors. Because the latter case implies that no structural change takes place, the only possibility for structural change together with the existence of a balanced growth path is θ being equal to unity. Then ψ is equal to zero and the model is identical to a one-sector Ramsey economy. Therefore the model is also saddle-path stable with a stationary solution implying balanced growth of the aggregates. NP derive some properties concerning structural change in the steady state and highlight the ability of the model to allow for a hump-shaped development of at least one sector. In the working paper version of their article NP also provide some numerical analyses with a calibrated model which imposes the steady state.

For our investigation of structural change among the three main sectors of the private economy this approach neglects one important fact: Structural change from the mid of the 19th century to our days is more a phenomenon of the transition towards the steady state rather than it is a phenomenon of the steady state itself. In the following we explore the role of structural change on the transition path to the steady state. This requires us to take the model as described above to the computer, to calibrate the parameters, and to use numerical techniques to trace out the transitional dynamics.

4 Numerical solution and model calibration

Since we are faced with a saddle-path stable equilibrium the usual numerical methods for solving non-linear differential equations are not applicable. The reason is that these methods solve the differential equations forward for some choice of initial values. If the differential equations are associated with saddle-path stable dynamics any deviations of the initial conditions from the (infinitely thin) saddle path are magnified and lead to trajectories that increasingly deviate from the equilibrium. Those deviations of the initial conditions cannot be avoided in numerical practice and thus no trajectory will lead into the equilibrium and the saddle-path cannot be traced out by forward-solving differential equations.

A simple and effective way out of this problem is suggested by Judd (1998, p. 357) and is known as reverse shooting. Reverse shooting proposes simply to multiply the differential equations to be solved by minus unity and then solving the system backwards by a usual ordinary differential equations solver from the steady-state with the steady-state position as the initial condition. The multiplication by minus unity reverses the direction of time, by that reverses the roles of the stable and the unstable loci of the saddle-path stable equilibrium and therefore enables tracing out the (formerly) unstable locus of the saddle-path. See Stemp and Herbert (2006) for a recent discussion of the application of numerical methods to solve economic models with saddle-path stable dynamics.

The specific ordinary differential equations solver we use is the function `lsoda()` of the package "odesolve" for the programming language R. This function provides an interface to the Fortran ODE solver of the same name (see the documentation of the R-package for details).

For the model in this paper we implemented the following solving strategy. First, we solved the dynamic equations for k_e and c_e (Eq. (19) and (20)) with normalizing the level of A_m to unity in the steady-state. For the productivity levels of the primary and the tertiary sector (A_a and A_s) we tried to find some reasonable values in the empirical literature with special attention to the initial (target) levels of employment shares. By doing so, the employment shares for the point in time, when balanced growth can be assumed to be reached are given by Eq. (15) and Eq. (16).[3]

For the parameter calibration, we mainly follow the commonly accepted values employed in economic growth theory that are consistent with empirical findings for the German and the U.S. economy. We also tried to stick to the values used in the working paper version of NP (Ngai and Pissarides, 2004) as close as possible.

In the Cobb-Douglas production function the capital share is commonly assumed to be about 30 percent (DeJong and Dave, 2007). Simon (1990) confirms this view and assumes the labor's share in production to be about two thirds in advanced economies. He argues that this share is roughly constant over time. Further, Jones (2003, table 1) shows that the capital share for the secondary and the tertiary sector are almost equal and about one third, respectively and Sokoloff (1986) confirmed this value about 0.3 for manufacturing in the early stages of industrialization. This is also consistent with the

[3]In a former draft we simply assumed that the levels for A_a and A_s are arbitrary numbers (also equal to unity). Accordingly, all other variables in the system were determined by the algorithm, such that n_a and n_s develop as described in Eq. (15) and n_m develops as described in Eq. (16) with x_a and x_s taken from Eq. (10) and x_m as the Numéraire. Finally, the income in efficiency units y_e develops as Eq. (14) which means $y_e = k_e^{\alpha}$.

Hence, we got the steady-state values of k_e and c_e as well as the values for n_a, n_m, and n_s. Due to the choice of A_a and A_s equal to unity, the sector shares at the steady state are equal for both sectors and therefore not the values expected in steady state. To correct that, the values of n_a, n_m, and n_s that are expected in steady state are chosen and the productivity levels are computed according to Eq. (10). These values and the solution values for k_e and c_e in the steady state in the former specification are used as initial values for the solution algorithm.

For the sector shares in the steady state, different assumptions are plausible. According to Fourastié (1949, 1969), the sectoral employment shares at the steady state should be around 10 percent for the primary and secondary sector, respectively, and about 80 percent for the tertiary sector. The NP model, however, implies that the share of the primary sector converges to zero, the share of the secondary sector converges to the savings rate and the share of the tertiary sector converges to one minus the savings rate. A third possibility would be to choose any point in time when the convergence to the steady-state may be assumed to be completed and take the realized data of the sectoral employment shares. In the former version of this paper we follow this second strategy and assume that the employment shares in the steady state are those which the model of NP predicts for convergence. Because a value of zero is not possible for the primary sector in the numerical investigation, we use 0.01 as a value very close to it. The tertiary sector is chosen to be close to one minus the saving rate and the secondary sector fulfills the requirement that all three sector shares have to add up to unity. This leads to the result that the secondary sector is producing only for capital accumulation also on the transition path to the steady state and the primary sectors share has been also unreasonable small also at the transition path.

data appendix and further results of Bernard and Jones (1996). Furthermore, Gomme and Rupert (2007) compute a value of about 0.283. So we decide to use $\alpha = 0.3$ in the baseline specification and vary from 0.25 to an upper bound of 0.4 which is the value Kongsamut et al. (2001) use in their numerical solution.

Following DeJong and Dave (2007), δ is in a range between 0.01 and 0.04 for the quarterly frequency, implying values of 0.04 up to 0.16 for annual frequency. King and Rebelo (1999) agree with these findings and point to a depreciation rate of about 10 percent. Gomme and Rupert (2007) use a value of 0.06 and NP use a value of 0.03 in their investigation. We use 0.05 in our baseline specification and vary from 0.025 to 0.1 to check for robustness.

The annual population growth rate for the whole period from 1850 until 2003 is about 1.66 percent for the U.S. and about 0.59 percent for Germany (Maddison, 2007). For the last ten years the growth rate for the U.S. became smaller than one percent and very close to zero for Germany. NP use an annual growth rate of population of two percent. In our baseline specification the population growth rate is one percent and we let the population growth rate vary from zero growth to four percent in the sensitivity analysis.

Concerning the utility function, the following parameter values seem to be appropriate. The time discount rate ρ is assumed to be between 0.01 and 0.02 in DeJong and Dave (2007, ch. 6). Gomme and Rupert (2007) provide a higher value of 0.065. We followed NP who use a value of 0.03 and tested for the range between 0.01 and 0.07. For θ a value of about two is taken to be realistic in economic literature and a value of unity is frequently used as a benchmark. Furthermore, Kongsamut et al. (2001) use a value of three in their calibration. We set $\theta = 1$ in the baseline specification and test for a range up to three. For the elasticity of substitution ε NP calculate a value around 0.3. We take this value for our baseline specification and test in a range between 0.1 and 0.5.

The weighting parameters ω_i are not explicitly set by NP and the empirical literature does not provide any values, either. Kongsamut et al. (2001) use values of 0.1 for the primary sector, 0.15 for the secondary sector, and 0.75 for the tertiary sector. We follow Kongsamut et al. (2001) in our baseline specification but test the robustness for the whole possible range in the following way with the restriction that they have to add up to unity.

For productivity, NP state that the productivity growth differences between agriculture and manufacturing as well as between manufacturing and services are about 0.01 percentage points (Historical Statistics of the United States, see NP, 2004). For their calculations they assume that prices are reflected in productivity only. Bernard and

Jones (1996) show that the growth rate of multi factor productivity of 14 OECD countries between 1970 and 1987 is on average 0.03 for agriculture, 0.02 for manufacturing, and 0.008 for services, respectively which supports the findings of NP. For an earlier period from 1800 until 1948 Kendrick (1961) shows that the TFP growth rate in agriculture is smaller than the TFP growth rate of manufacturing in the U.S. We decide to use the values provided by NP in the baseline specification and test for an alternative with converging sectoral TFP growth rates in section 6.

Regarding the productivity levels, Bernard and Jones (1996) report in their data appendix that manufacturing has a higher TFP level than services. While the TFP level in 1987 was more than five times higher in manufacturing than in services for the U.S. it has been about three times as high in Germany. We set the value to $A_s = 0.25$ which is in between both empirical values for the year 1987. For the ratio between agriculture and manufacturing no values could be found, so we follow Kongsamut et al. (2001) and set A_a four times higher than A_m which means $A_a = 4$.

In sum, we perform our investigations with a baseline specification using the following parameter values reported in table 1. Min. and Max. indicate the ranges for the robustness assessment.

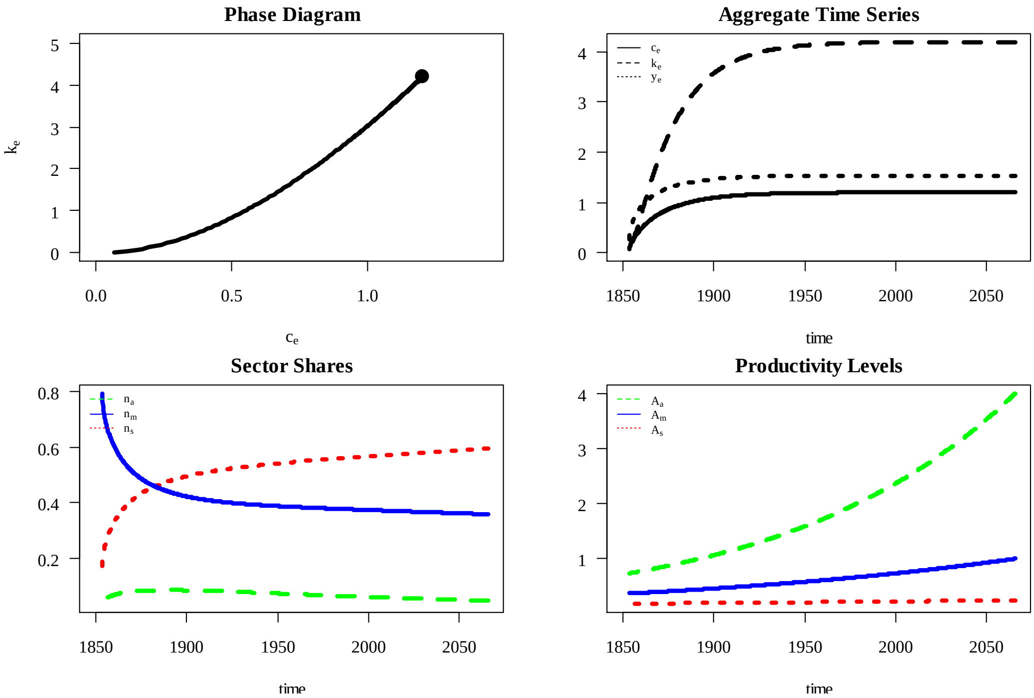
Table 1: Parameter values for the numerical solution of the original model

Description		baseline model	Min.	Max.
Share of capital	α	0.3	0.25	0.4
Depreciation rate	δ	0.05	0.025	0.1
Population growth rate	ν	0.01	0	0.04
Reciprocal elasticity of substitution	θ	1	1	3
Time discount rate	ρ	0.03	0.01	0.07
Elasticity of substitution across goods	ε	0.3	0.1	0.5
Weight of good in aggregate consumption				
Primary	ω_a	0.1	0.1	0.8
Secondary	ω_m	0.15	0.1	0.8
Tertiary	ω_s	0.75	0.1	0.8
Initial (target) TFP levels				
Primary	A_a	4		
Secondary	A_m	1		
Tertiary	A_s	0.25		
TFP growth rates				
Primary	γ_a	0.024	0.014	0.14
Secondary	γ_m	0.014		
Tertiary	γ_s	0.004	0.0014	0.014

5 Numerical results

This section presents the results of the numerical investigation of the NP model. The model has been solved as described in section 4. We start the discussion with the results obtained with the preferred parameter values and then proceed to a sensitivity analysis of these parameter values. Figure 3 presents the development of important time series of the model.

Figure 3: Economic development in the baseline specification



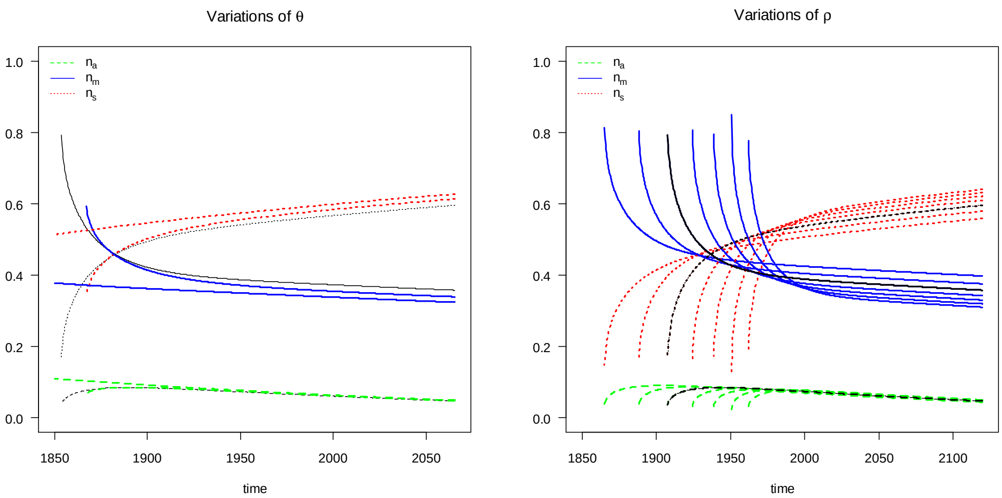
In the upper left panel the transition path towards the steady-state as marked by the bold dot is depicted in the $c_e - k_e-$ space. The curve is the saddle path computed by solving the original NP model backwards in time for the baseline parameters. The aggregate time series in the upper right panel show the development of national per capita income, per capita consumption, and the capital-labor ratio in efficiency units, respectively. As it can be seen in the figure, these values confirm the ranges found by empirical research.[4]

[4]The overall growth rate of GDP had been 2.03 percent and 1.99 percent for the U.S. and Germany, respectively (Maddison, 2007). Further, Simon (1990) stated that the capital-output ratio using gross data is about 2.8 for the year 1989. The average saving rate from 1850 until 1988 has been 0.1807 for the U.S. and Germany shows an average of 0.2617 from 1950 until 1988 (Maddison, 1992, table 2). This is consistent with the findings for the total investment of share of GDP published in Jones (1995), table 3. Here Germany's share was about 23 percent in the late 1980s and the share of the U.S. about 18 percent. For the more recent years the Bureau of Economic Analysis states a value between 18 and 20 percent for the U.S. economy in their main aggregate variables compiled from the OECD questionnaires and the German council of economic experts states a value of about 20 percent for Germany (Sachverständigenrat zur Begutachtung der gesamtwirtschaftlichen Entwicklung, 2005).

The bottom right panel shows the development of the productivity levels and the left bottom panel shows the trajectories of the employment shares of the three sectors. We observe that the sectoral development does not reproduce the historical development. The share of the primary sector is initially very small and increases in the first years, the secondary sector decreases monotonically and the tertiary sector increases monotonically. Hence, the original NP model is not able to replicate the hump-shaped development of the secondary sector. Given that the marginal productivity of capital is high at the beginning, providing a large incentive to accumulate capital, and furthermore given that share of the secondary sector cannot be lower than the savings rate (one minus the consumption ratio) the share of the secondary sector is initially large by construction. Lowering this initial share requires a substantial modification of the model as discussed below.

In the following figures we report the results of the analysis for different parameter constellations. Naturally, these variations also affect the aggregate time series. Because of space limitations on the one hand and only minor differences of the results on the other hand here we only discuss the effects on the sector shares. The aggregate development is quite the same with either slower or faster transition to the steady state and only the levels of consumption per capital in efficiency units, the capital-labor ratio in efficiency units and therefore also the ratio between capital and output varying. Figure 4 shows the results of variations for parameters θ and ρ of the utility function.

Figure 4: Variation of preference parameter values θ and ρ



The left graph presents the sectoral development when θ equals one, two, or three, respectively. For θ equal to unity the development is most dynamic and the transition to balanced growth is the fastest. As θ increases the dynamics are smoother and the transition becomes slower. Furthermore, a higher value of θ results in a lower saving rate and therefore in a lower steady state employment share of the secondary sector. In the right graph of the figure the results for varying values of ρ in the range between 0.01 and 0.07 show that higher values of ρ result in a faster transition and a lower savings rate. In sum, the basic pattern found for the baseline parameter setting appears to be robust.

Figure 5 shows the results, for variations of the values of parameters ε and ω. The left graph presents the results for varying values of ε between 0.1 and 0.5. A higher value of ε results in a slightly higher steady state employment share of the secondary sector associated with a slightly higher saving rate. The speed of transition does not change at all. The right graph of the figure 5 presents the results for different parameter settings of the sectoral ω_i which again shows little variation in the dynamics of the sectoral employment. The values are chosen in a range between 0.1 and 0.8 for all ω_i with the restriction that the sum of ω_i has to be equal to unity. Changes are mainly in the levels of the sector shares of the secondary and the tertiary sector.

Figure 5: Variation of preference parameter values ε and ω_i

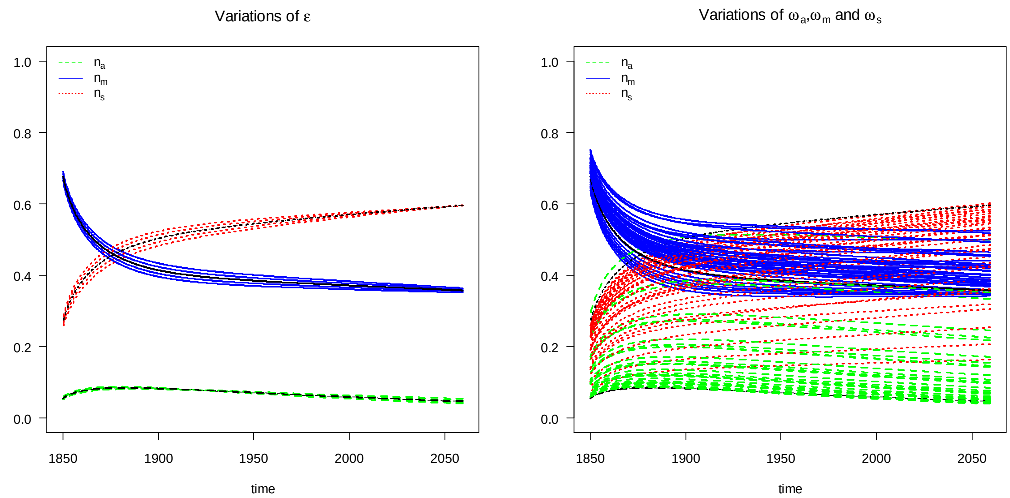
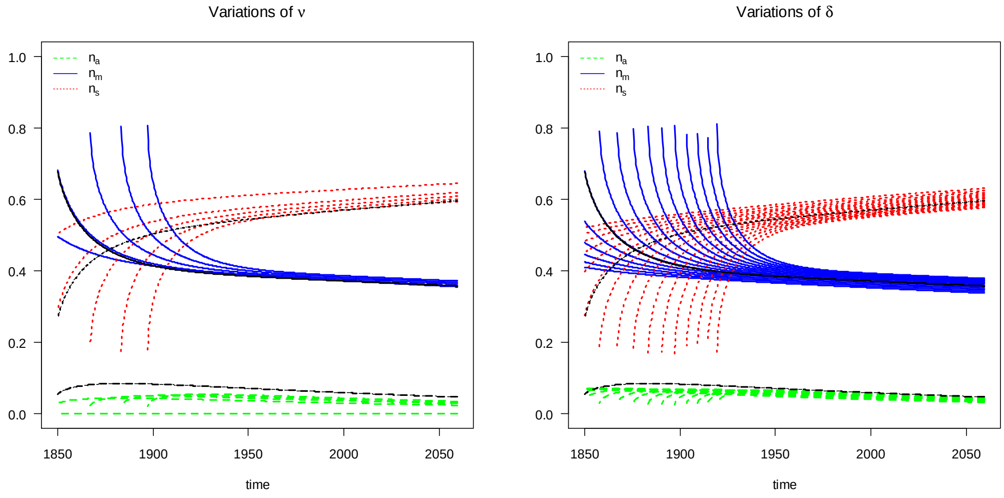


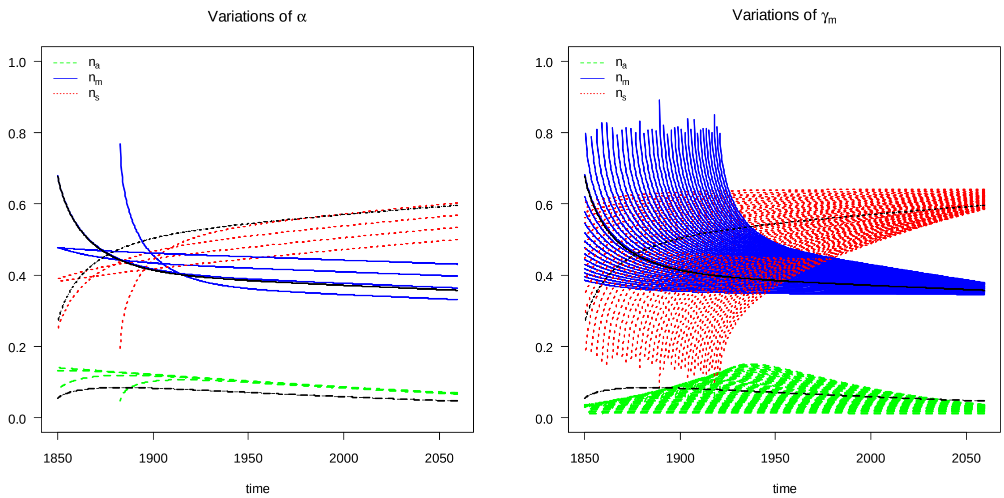
Figure 6 presents the sensitivity analysis results for the parameter values of the working force growth rate (left hand side) and the depreciation rate (right hand side). We let ν vary between zero and four percent and δ between two and a half and ten percent. As before, this parameter variation does not change the general pattern of the sectoral employment shares. Again, the shares of the secondary and the tertiary sector vary as well as the speed of transition does. In these cases a higher rate of population growth as well as a higher depreciation rate result in a faster transition with a higher share of the secondary sector and a higher savings rate, respectively.

Figure 6: Variation of the population growth rate ν and the depreciation rate δ



In figure 7 the results for a varying capital share in the production function α in the range from 0.25 to 0.4 on the left hand side and the different productivity growth rates for the secondary sector γ_m in the range from 0.001 to 0.05 are reported. The graphs show that a larger α results in slower dynamics and a lower share of the tertiary sector for the economy in steady state. Faster productivity growth in the secondary sector leads to a faster transition to the steady state simultaneously with a higher share of the secondary sector's employment. Besides that, these parameter variations are again not suited to generate different dynamics for the sectors.

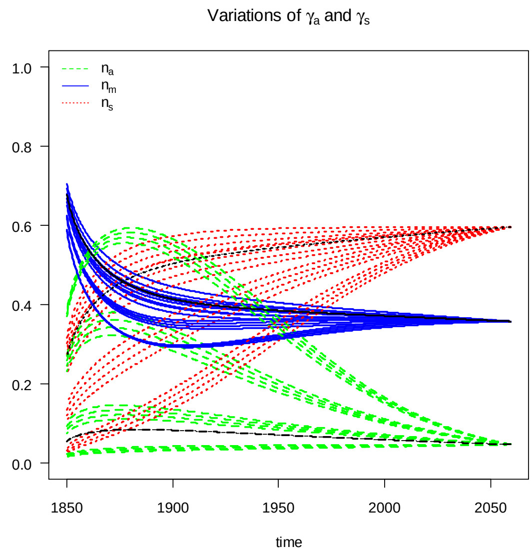
Figure 7: Variation of the capital share α and productivity growth rate γ_m



As NP emphasize, the dynamics of the sectoral development in their model are mainly driven by sectoral differences in productivity growth. Therefore, we investigate the effect of different sectoral productivity growth rates on the sector share dynamics.

The results can be found in figure 8. The development of the sectors in the baseline parameter settings are plotted in black for all three sectors (as above the development of the primary sector is plotted by a dashed line, the development of the tertiary sector by a dotted line, and the development of the secondary sector by a solid line). We let the productivity growth rate of the primary sector γ_a vary in an interval from 0.014 to 0.14, meaning that we have a relative productivity growth rates of one to twenty times of the growth rate of the secondary sector. The productivity growth rate of the tertiary sector γ_s is varied in a range from 0.0014 to 0.014, meaning that the productivity growth of the secondary sector was between one tenth and twice the value of the secondary sector. Because only the relation of the different TFP growth rates between the three sectors is of interest the TFP growth rate of the secondary sector was fixed to the value of 1.4 percent used for the baseline specification. As long as γ_a is smaller than five times γ_m the dynamic does not change very much. Beginning with a value of γ_a that equals five times γ_m the share of the primary sector becomes dynamic. As can be seen from the figure, now the share of the primary sector increases first at the expense of the tertiary sector and decreases over time later on. It thus shows the hump-shaped pattern that characterizes the secondary sector in the data. Once γ_a is ten times γ_m the curvature of the tertiary sector share becomes s-shaped and the maximum share of the primary sector increases to 60 percent. This shows that the model has the potential to show a highly dynamic development between the sector shares. But in this specification the realized time paths are not replicable and even more the dynamics are only observable when the growth differentials become exorbitantly high.

Figure 8: Variation of productivity growth rate differences



Summarizing the main findings of this section, we have to admit that the original NP model has major problems in replicating the sectoral development of the three main sectors of an economy on the transition path. In the model differential productivity growth rates are the main drivers of structural change. Our results show that this is indeed the case, but the dynamics do not even qualitatively replicate the historical development. In particular, the secondary sector's share is decreasing monotonically as time goes by and the primary sector is first increasing and decreasing later on which then replicates a hump-shaped development of this sector. However, this result for the primary sector can only be achieved for very large differences in the productivity growth of the primary and the secondary sector, which cannot be verified with historical data. In the next section the model will be extended to endogenous growth and possible improvements to the sectoral development are investigated.

6 Asymptotic converging TFP growth rates

NP choose a way of modeling sector specific technological change (Eq. (8)) by assuming that the productivity growth rates γ_i are constant values which only differ in their magnitude across sectors. This kind of modeling is necessary, due to the fact that the aim of their investigation is to show that productivity driven structural change is consistent with balanced growth. As already mentioned, structural change and aggregate economic growth can be consistent, but for the economic development in the light of the three-sector hypothesis, the assumption that economic growth has been at steady-state growth for the last 150 to 200 years seems not to be very realistic. Furthermore, the empirical literature clearly shows that the assumption of a higher productivity growth rate in the primary sector compared to the secondary sector for the whole time period does not hold. A summary of various estimates of TFP growth of the primary and secondary sectors in the U.S. can be found in tables 2 and 3, respectively.

Tables 2 and 3 give a rough overview about the sectoral productivity development. We are aware that the comparability between different sources is limited since the assumptions for computing TFP growth rates differed greatly cross these sources.[5] In fact, as NP assumed, the growth rate was larger in the primary sector than in the manufacturing sector for some time after World War II, but as one can see from the tables, productivity growth in the early stages of economic development in the U.S. was averagely larger in secondary than in the primary sector.

For the case of the tertiary sector no data for such a long time period is available. We only can rely on data for the shorter time span after World War II. As already mentioned above by Bernard and Jones (1996) the productivity growth rate averagely

[5] See the data appendix of Dennis and Iscan (2009) for a detailed discussion on this issue.

Table 2: Annualized productivity growth rates of the primary sector (percent)

<table>
<tr><th>Period</th><th>G</th><th>M</th><th>CW</th><th colspan="2">K</th><th colspan="2">USDA</th></tr>
<tr><td>1800-1810</td><td>-0.30</td><td rowspan="4">0.19</td><td rowspan="4"></td><td></td><td></td><td></td><td></td></tr>
<tr><td>1810-1820</td><td>0.36</td><td></td><td></td><td></td><td></td></tr>
<tr><td>1820-1830</td><td>1.11</td><td></td><td></td><td></td><td></td></tr>
<tr><td>1830-1840</td><td>1.40</td><td></td><td></td><td></td><td></td></tr>
<tr><td>1840-1850</td><td>-0.40</td><td rowspan="4">0.56</td><td rowspan="2">-0.14</td><td></td><td></td><td></td><td></td></tr>
<tr><td>1850-1860</td><td>0.44</td><td></td><td></td><td></td><td></td></tr>
<tr><td>1860-1870</td><td>0.66</td><td>1.47</td><td></td><td></td><td></td><td></td></tr>
<tr><td>1870-1880</td><td>1.34</td><td rowspan="3">0.52</td><td>1869-1879</td><td>1.46</td><td></td><td></td></tr>
<tr><td>1880-1890</td><td>0.38</td><td rowspan="2">0.15-
0.56</td><td>1879-1889</td><td>0.54</td><td></td><td></td></tr>
<tr><td>1890-1900</td><td>0.62</td><td>1889-1899</td><td>1.05</td><td></td><td></td></tr>
<tr><td>1900-1910</td><td></td><td></td><td></td><td>1899-1909</td><td>-0.24</td><td></td><td></td></tr>
<tr><td>1910-1920</td><td></td><td></td><td></td><td>1909-1919</td><td>-0.28</td><td></td><td></td></tr>
<tr><td>1920-1930</td><td></td><td></td><td></td><td>1919-1929</td><td>1.24</td><td></td><td></td></tr>
<tr><td>1930-1940</td><td></td><td></td><td></td><td>1929-1937</td><td>0.80</td><td></td><td></td></tr>
<tr><td>1940-1950</td><td></td><td></td><td></td><td>1937-1948</td><td>2.69</td><td></td><td></td></tr>
<tr><td>1950-1960</td><td></td><td></td><td></td><td>1948-1953</td><td>3.69</td><td rowspan="2">1949-1973</td><td rowspan="2">0.15-
0.56</td></tr>
<tr><td>1960-1970</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>1970-1980</td><td></td><td></td><td></td><td></td><td></td><td>1973-1979</td><td>0.76</td></tr>
<tr><td>1980-1990</td><td></td><td></td><td></td><td></td><td></td><td>1979-1990</td><td>2.46</td></tr>
<tr><td>1990-2000</td><td></td><td></td><td></td><td></td><td></td><td>1990-1996</td><td>1.65</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td><td>1987-1999</td><td>1.58</td></tr>
<tr><td>2000-2010</td><td></td><td></td><td></td><td></td><td></td><td>1999-2007</td><td>1.23</td></tr>
</table>

Sources: (G)-Gallman (1972), table 8; (M) - Mundlak (2005), table 2 ; (CW) - Craig and Weiss (2000), table 3; (K) - Kendrick (1961), table B-I; (USDA) - U.S. Department of Agriculture, table 1: Indices of farm output, input, and total factor productivity for the United States, 1948-2008, on the web: http://www.ers.usda.gov/Data/AgProductivity/.

was about one percentage point higher in the secondary than in the tertiary sector for the last 40 years. For the long run we get only hints in the empirical literature that productivity growth in services was smaller than productivity growth in manufacturing. For example, Fuchs (1968, pp. 75-76) supports the assumption that productivity growth in the service sector lags behind that of manufacturing on average for the period from 1929 to 1965.

Therefore, in our investigation we modeled sector specific exogenous technological change in an alternative way. As in the original model TFP in the manufacturing sector grows at constant rate γ_m and the growth rates of the primary and the tertiary sector are initially higher or lower. But in contrast to the original model, TFP growth of the primary and the tertiary sector are related to the growth rate of the secondary sector which can be interpreted in terms of spillover effects originating from the sec-

Table 3: Annualized productivity growth rates of the secondary sector (percent)

Period	S	K		GH		BLS	
1800-1810							
1810-1820							
1820-1830	**3.03**						
1830-1840	**1.32**						
1840-1850							
1850-1860	**2.43**						
1860-1870							
1870-1880		1869-1879	**0.87**				
1880-1890		1879-1889	**1.96**				
1890-1900		1889-1899	**1.12**				
1900-1910		1899-1909	**0.72**				
1910-1920		1909-1919	**0.29**				
1920-1930		1919-1929	**5.31**				
1930-1940		1929-1937	**1.95**				
1940-1950		1937-1948	**1.56**				
1950-1960		1948-1953	**2.53**	1949-1973	**1.50**		
1960-1970							
1970-1980				1973-1979	**-0.40**		
1980-1990				1979-1990	**1.00**		
1990-2000				1990-1996	**1.90**	1987-1999	**1.09**
2000-2010						1999-2007	**2.31**

Sources: (S) - Sokoloff (1986), table 13.9, (K) - Kendrick (1961), table A-XXIII; (GH) - Gullickson and Harper (1999), table3; (BLS) - Bureau of Labor Statistics, table: Aggregate Manufacturing and Manufacturing Industries KLEMS Multifactor Productivity Tables, on the web: http://www.bls.gov/mfp/mprdload.htm.

ondary sector. The new equations for the primary and the tertiary sector TFP growth rates are now

$$\frac{\dot{A}_i}{A_i} = \gamma_m \exp\left(\frac{\gamma_i}{A_i}\right) \text{ for } i = a, s. \tag{21}$$

Here the γ-parameters of the primary and the tertiary sector have to be interpreted in a different way. As the productivity level A of sector i increases, the fraction γ_i/A_i converges to zero and the productivity growth rate of sector i converges to the productivity growth rate of the manufacturing sector.[6] If the parameter γ_i is negative, the fraction γ_i/A_i is negative and therefore $\exp(\gamma_i/A_i)$ is smaller than unity and hence converges to unity from below. The TFP growth rate of sector i then is smaller than the TFP growth rate of the secondary sector. For positive values of γ_i the TFP growth rate of

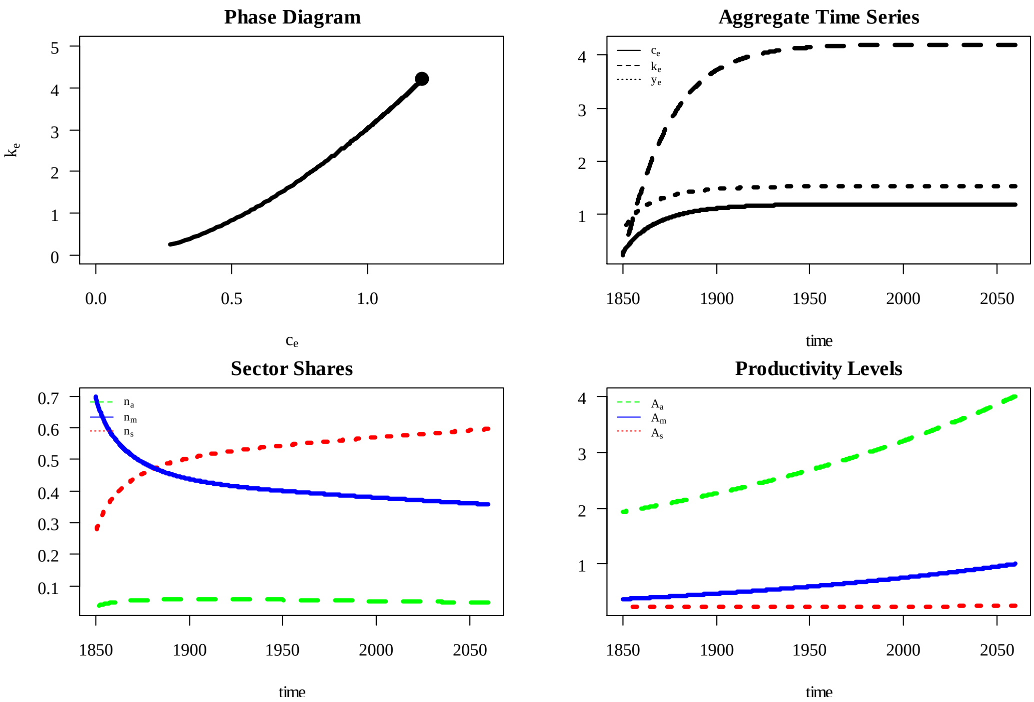
[6]As NP show, the necessary condition for a balanced growth path is that ψ is constant. That is reached in the original model by setting θ equal to unity. In this specification the productivity growth rates converge and therefore the term $(\gamma_m - \bar{\gamma})$ converges to zero. With this new specification a balanced growth path can be reached and supply-side driven structural change can take place on the transition path, when $\theta \neq 1$.

sector i is larger than the TFP growth rate in the secondary sector and the growth rate is converging from above. For the numerical exploration we used the following parameter values.

Table 4: Additional parameter values (converging productivity growth rates)

Description		baseline model	Min.	Max.
TFP growth rate parameters				
Primary	γ_a	-0.8	-0.1	-0.9
Secondary	γ_m	0.014		
Tertiary	γ_s	-0.8	-0.1	-0.9

Figure 9: Economic development with converging productivity growth rates

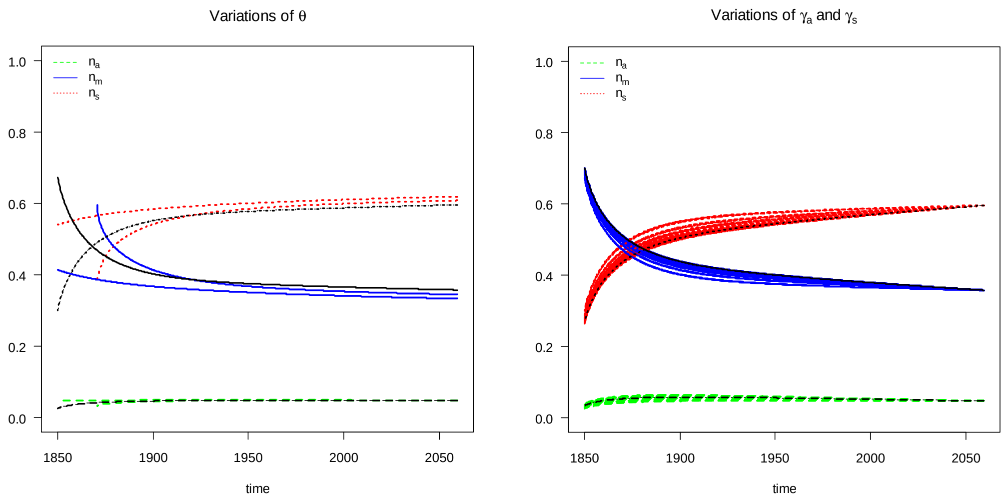


The model solution in the baseline specification with $\theta = 1$ can be seen in figure 9. Despite a smaller productivity growth rate of the primary sector [7] the development of the aggregate time series as well as the development of the three sectors change only little. The tertiary sector increases monotonically as the stylized facts suggest. But again, the primary sector share is very small right from the beginning and the sector share of the secondary sector is very large at the beginning and decreases monotonically. This is due to the high savings rate at the early stages of development in this model.

[7]The initial value of the productivity level is now more than two times as high as in the baseline specification of the original model (see figure 3).

Therefore, we used the parameters from the baseline specification 1 but varied again for θ from 1 to 3. But as the left panel in figure 10 reflects this variation does not yield better results. The right panel of figure 10 shows that variations in the parameters γ_a and γ_s do not change the basic findings.

Figure 10: Variations of θ and γ_a and γ_s



In sum, the introduction of an alternative way to model sector specific TFP growth rates does not change the main findings of the numerical explorations of the original model. The model still has the same shortcomings in the ability of replicating the stylized facts of the long-run sectoral development. Therefore, the next section turns to a more detailed consideration of demand-side effects.

7 Non-homothetic Preferences

Up to now the process of structural change is only driven by the supply side, namely the differential rates of technological progress in the three sectors of the economy. Thus, it may be beneficial to introduce a source of structural change also from the demand side with the aim of letting structural change appear to be more pronounced and closer to the empirical facts. The usual way to model structural change from the demand side is by non-homothetic preferences (see e.g., Kongsamut et al., 2001; Bonatti and Felice, 2008). In the context of the NP model this can be easily achieved by replacing the CES aggregator function for the aggregation of the consumption quantities of the three sectors in equation (9) of NP by a S-Branch Utility function (Brown and Heien,

1972) for the special case of three sectors and only one consumption good belonging to each sector

$$\varphi\left(c_a, c_m, c_s\right) = \left(\omega_a\left(c_a + \bar{c}_a\right)^{(\varepsilon-1)/\varepsilon} + \omega_m\left(c_m + \bar{c}_m\right)^{(\varepsilon-1)/\varepsilon} + \omega_s\left(c_s + \bar{c}_s\right)^{(\varepsilon-1)/\varepsilon}\right)^{\varepsilon/(1-\varepsilon)} \tag{22}$$

with $\bar{c}_i,\ i \in \{a, m, s\}$ as sector specific constants that represent subsistence levels of the sectors. As in the homothetic case of the CES aggregator function, $\varepsilon > 0$ is the intertemporal elasticity of substitution and the ω_i are weighting parameters with $\omega_i > 0\ \forall i$ and $\sum \omega_i = 1$. This new aggregator function nests the two utility functions of NP and Kongsamut et al. (2001). We get the special case of NP if the sector specific constants $\bar{c}_i$ equal zero and the ε is smaller than unity. On the other hand the preferences used by Kongsamut et al. (2001) are the special case in which $\varepsilon = 1$ (which results in a Stone-Geary form of utility (Geary, 1951; Stone, 1954)) and $\bar{c}_a < 0$, $\bar{c}_s > 0$, and $\bar{c}_m = 0$.[8]

For the static efficiency conditions we get

$$\frac{p_m}{p_i} = \frac{A_m}{A_i} = \frac{\omega_i}{\omega_m}\left(\frac{c_m + \bar{c}_m}{c_i + \bar{c}_i}\right)^{1/\varepsilon}. \tag{23}$$

Imposing the condition that $\bar{c}_m = 0$ and rearranging for c_i/c_m results in

$$\frac{c_i}{c_m} = \left(\frac{A_i}{A_m}\frac{\omega_i}{\omega_m}\right)^{\varepsilon} - \frac{\bar{c}_i}{c_m} \text{ and } \frac{A_m}{A_i}\frac{\omega_m}{\omega_i} = \left(\frac{c_m}{c_i + \bar{c}_i}\right)^{1/\varepsilon}. \tag{24}$$

For the ratio of consumption expenditure on good i to consumption expenditure on the manufacturing good x_i we obtain now:

$$x_i = \frac{p_i c_i}{p_m c_m} = \left(\frac{A_m}{A_i}\right)^{1-\varepsilon}\left(\frac{\omega_i}{\omega_m}\right)^{\varepsilon} - \frac{A_m}{A_i}\frac{\bar{c}_i}{c_m}. \tag{25}$$

Since these two differential equations can no longer be solved analytically for the steady-state position we here also have to use a numerical method. We choose Broyden's method which works quite satisfactorily in our case.[9] This specification of the aggregator function also requires a numerical solution for the first-order conditions which has to be integrated into the whole solution procedure. Therefore, we implement the Gauss-Seidel algorithm in the numerical solution which works sufficiently well in our case. For parameter calibration table 5 reports the values used for the subsistence terms. As already mentioned above, we set $\bar{c}_m = 0$. Furthermore, we use $\bar{c}_s = 0$ to avoid home production and to see, whether the Baumol effect is sufficient to replicate

[8] We follow Kongsamut et al. (2001) and restrict $\bar{c}_m$ to equal zero.

[9] See Judd (1998, p. 244) for a description of this method.

the dynamics. Alternative computations with $\bar{c}_s \neq 0$ show that the ability of the model to replicate the empirical facts gets worse. The values for the the other parameters are those of the baseline specification.

Table 5: Additional parameter values (non-homothetic preferences)

Description		baseline model	Min.	Max.
Subsistence terms				
Primary	$\bar{c}_a$	-0.25	-0.4	-0.1
Secondary	$\bar{c}_m$	0		
Tertiary	$\bar{c}_s$	0	0	0.1

Figure 11: Economic development with non-homothetic preferences ($\theta = 1$)

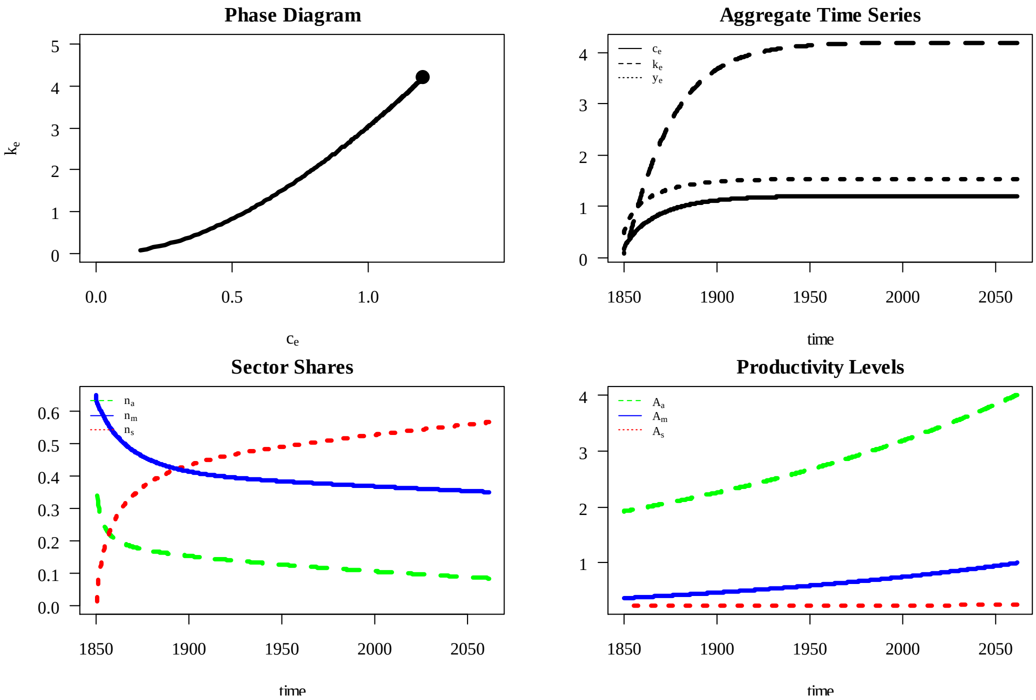
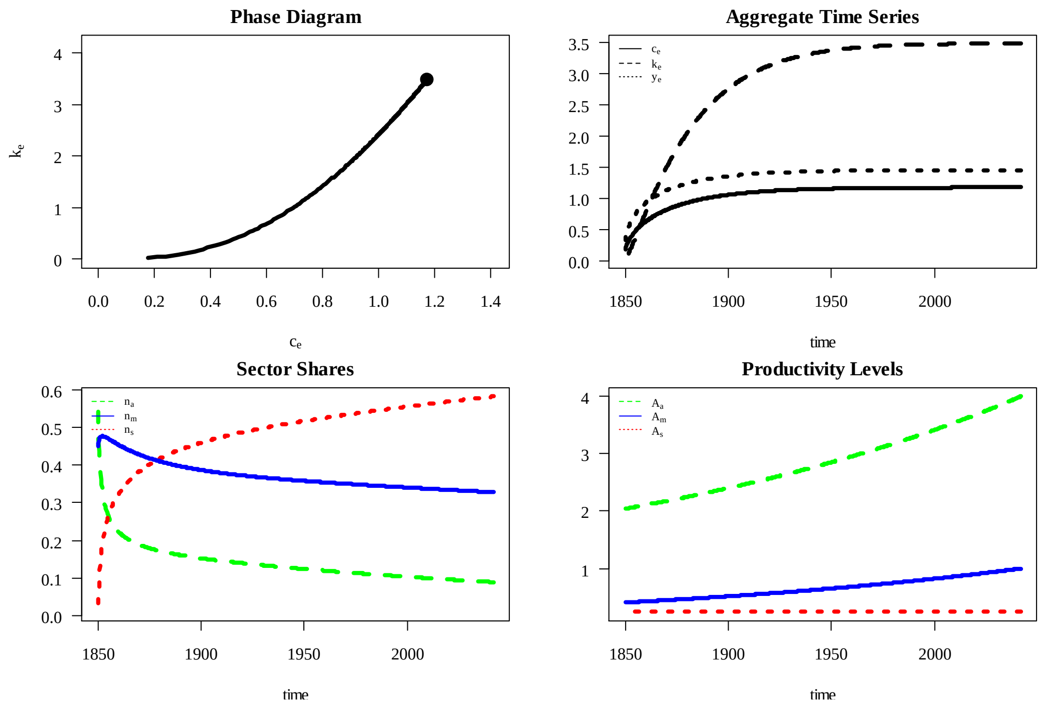


Figure 11 shows that the aggregate development and the development of sector specific TFP does not change compared to the findings of the previous section. In contrast to the case with homothetic preferences, the development of the tertiary sector and most notably the development of the primary sector share changes considerably. While the primary sector share was initially small in the former model specification, the introduction of non-homothetic preferences results in a sector share of more than thirty percent in the starting period which then decreases to about ten percent in the course of time. Comparing figures 3, 9, and 11 it appears that the higher share of the primary sector in early stages of development is only at the expense of the tertiary sector share.

Moreover, the development of the secondary sector share is very similar to the results found in the previous section. As discussed above, this might be due to the fact of a very high savings rate in the early development in this model specification. The alternative developments with $\theta = 2$ and $\theta = 3$ can be found in figures 12 and 13.

Figure 12: Economic development with non-homothetic preferences ($\theta = 2$)

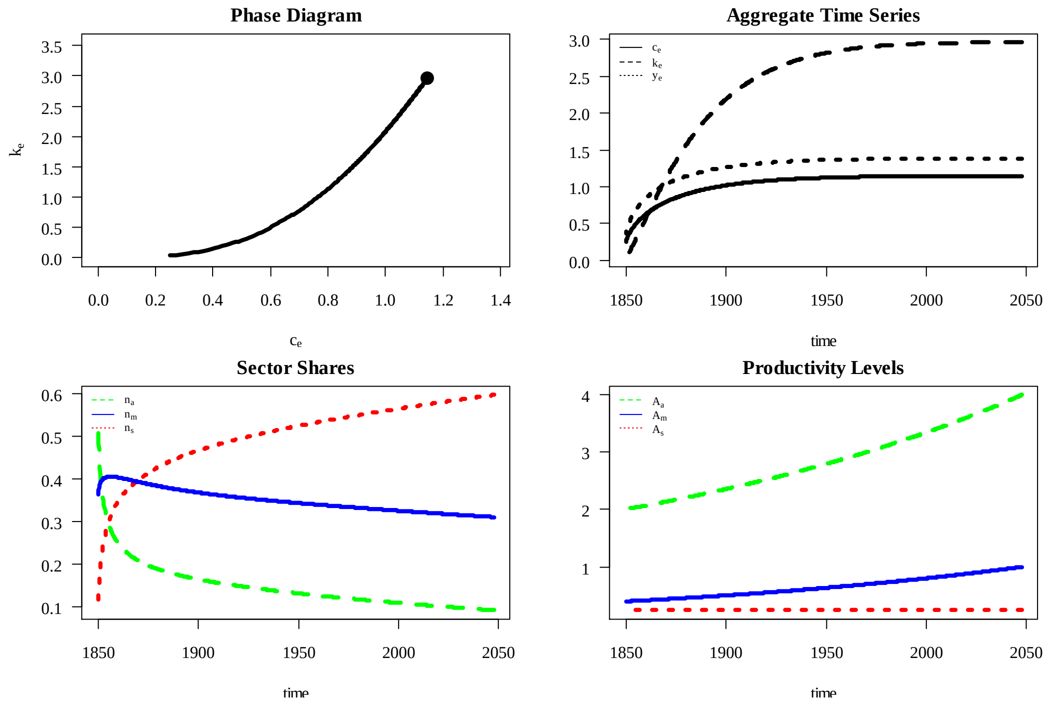


A higher value of $\theta = 2$ and even more $\theta = 3$ changes the results substantially. In this specification structural change in the long run is replicated quite well even though only in a very stylized way. The model does not only explain the declining share of the primary sector and the rising share of the tertiary sector but most notably also the hump-shaped development of the secondary sector in early stages of development. Comparing figures 12 and 13 shows that a higher value of θ leads to an initially lower share of the secondary sector and a distinct characteristic development.

Besides, in the panels of the aggregate time series and the phase diagram a different development for capital accumulation can be seen clearly. While the capital intensity (k_e) grows right from the beginning for $\theta = 1$ (figure 11) the growth starts with some lag in time if $\theta = 2$ and even more if $\theta = 3$ (figures 12 and 13, respectively). All this is of course a straightforward consequence of a lower elasticity of inter-temporal substitution $1/\theta$ which is associated with more consumption today and thus lower savings.

This picture changes only little for variations of the subsistence parameter $\bar{c}_a$ as can be seen in the left panel of figure 14 where we use the last model specification with $\theta = 3$ and vary $\bar{c}_a$ between 0.1 and 0.4.

Figure 13: Economic development with non-homothetic preferences ($\theta = 3$)



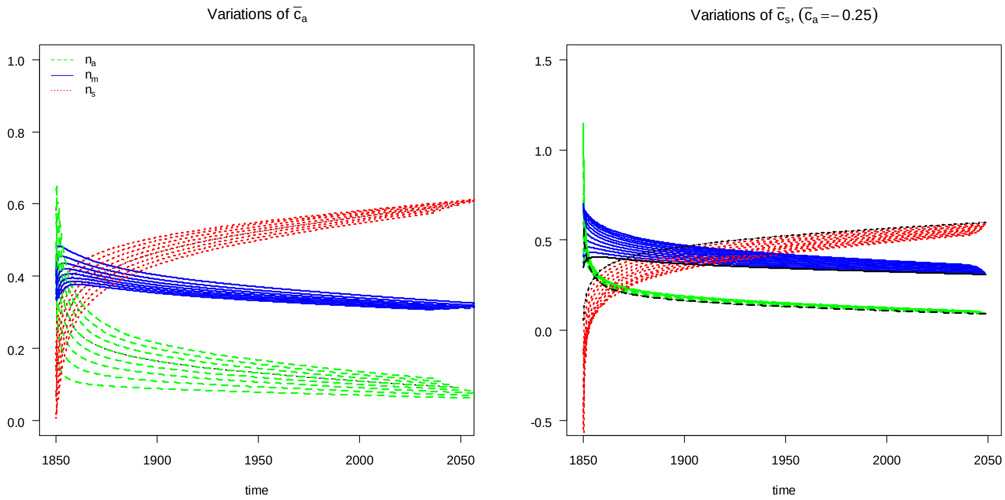
When $\bar{c}_a$ gets larger the decline of the primary sector share is slower and the level is higher over the whole period. The shares of the secondary and the tertiary sector become smaller and moreover the hump-shaped development of the secondary sector is less pronounced and the transition to the steady state is also faster.

Once, $\bar{c}_s \neq 0$ is introduced, the sectoral development changes dramatically and worsens the model's ability to replicate the stylized development as can be inferred in the right panel of figure 14. The level of the tertiary sector share becomes lower for the whole period, which is not overly surprising since this parameter can be interpreted as home production, which means that less labor is needed for market production for service goods. However, the model does not constrain the shares of the sector to be larger than zero and here the share of the tertiary sector becomes smaller than zero in the early stages of economic development, which is inconsistent with the realized economic development. This fact becomes stronger the larger $\bar{c}_s$ is chosen. Also, the hump-shaped development of the secondary sector is lost once a certain threshold for $\bar{c}_s$ is reached.[10]

The results so far show that the long-run sectoral development is determined by demand side as well as supply-side effects in this model. For the early stage of development (industrialization) non-homothetic preferences are the main source of structural change.

[10]In this case with $\bar{c}_a = -0.25$ a value of $\bar{c}_s = 0.03$ is sufficient.

Figure 14: Variation of the subsistence parameters $\bar{c}_a$ and $\bar{c}_s$



In the later phase of deindustrialization TFP growth differences between the sectors seem to be the main driver of structural change.

Despite the fact that the introduction of non-homothetic preferences and variations of θ leads to a better replication of the stylized development it holds that the share of the secondary sector remains quite large. This circumstance is due to the fact that the employment share of the secondary sector in which capital is accumulated is bounded from below by the savings rate $1 - c/y$ (see equation (14) of NP). This bound could be lowered by decreasing the inter-temporal elasticity of substitution but it still remains large for common values of θ. Introducing human capital accumulation into the model may be another means and this next step is persued in section 9.

Before that, we recall that a substantial weakness of neoclassical growth theory is the exogenously defined TFP growth. Therefore, we investigate whether our findings are consistent with endogenous technological change by introducing a very simple version of endogenous technological change in section 8.

8 Endogenous technological change

To endogenize technological change we simply assume that the economy invests a fixed share $0 < \eta < 1$ of aggregate output in activities related to research and development. This is justified by the almost constant share of research and development in GDP over the most recent two or three decades. This share is bounded in the range of 2 to 2.5 percent (data obtained from NSF, 2007, table 13). Stated in terms of efficiency units this investment is

$$R_e = \eta y_e = \eta k_e^\alpha \tag{26}$$

and its effect on productivity growth in the secondary sector is assumed to be

$$\dot{A}_m = A_m \gamma_m R_e, \;\; \gamma_m > 0. \tag{27}$$

The other two sectors benefit from the productivity growth generated in manufacturing according to

$$\dot{A}_i = A_i \gamma_m R_e \exp\left(\frac{\gamma_i}{A_i}\right), \;\; i = a, s. \tag{28}$$

Note that the γ-parameters now have a different role. They reflect differences in productivity growth rather than the productivity growth rates itself for the primary and the tertiary sector, respectively and the parameter value γ_m is simply a scaling constant. Hence, these parameters have to be interpreted in a different way as in the original NP model and this also has to be taken into account in the calibration.

Only slightly modified are the equations for $\bar{\gamma}$ and ψ which become

$$\bar{\gamma} = \left(x_a \dot{A}_a / A_a + x_m \dot{A}_m / A_m + x_s \dot{A}_s / A_s\right) / X \tag{29}$$

$$\psi = (\theta - 1)\left(\gamma_m \eta k_e^{\alpha} - \bar{\gamma}\right). \tag{30}$$

The differential equations for capital accumulation and consumption become

$$\begin{aligned} \dot{k}_e &= k_e^{\alpha} - c_e - R_e - \left(\gamma_m R_e / (1-\alpha) + \delta + \nu\right) k_e \\ &= (1-\eta)\, k_e^{\alpha} - c_e - \left(\gamma_m \eta k_e^{\alpha} / (1-\alpha) + \delta + \nu\right) k_e \end{aligned} \tag{31}$$

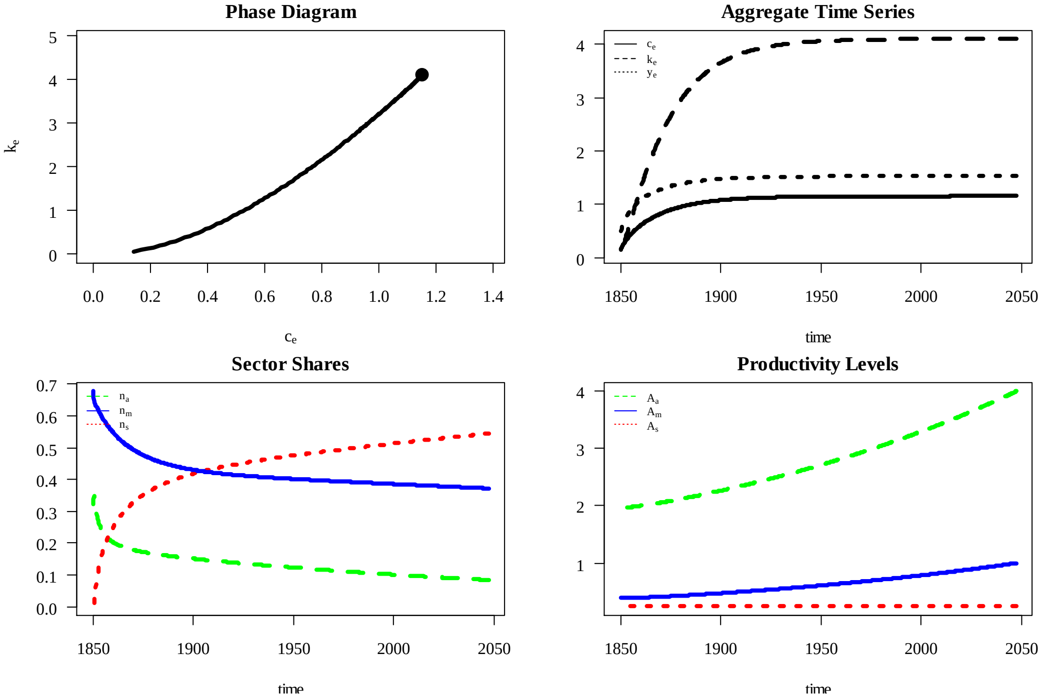
$$\dot{c}_e = c_e \left[\alpha k_e^{\alpha-1} + \psi - (\delta + \rho + \nu)\right] / \theta - \gamma_m \eta k_e^{\alpha} / (1-\alpha). \tag{32}$$

Eq. (31) reflects that research and development is assumed to take place in the secondary (manufacturing) sector which reduces current period consumption possibilities for the goods of this sector in addition to the reduction from the accumulation of physical capital. In this specification we use the parameter values of the baseline specification as described above and a value for η of 0.025 .

As figure 15 shows, the general development of the economy is very close to that in the purely neoclassical framework in section 5.

The economy-wide growth rate converges to two percent and the productivity levels are similar to those in the in steady state of the original model. The only difference shown in these series is that the growth rates of productivity are not any longer constant over time but grow as total GDP grows. Further, the capital-income ratio and the consumption share of total income lie in the ranges found in the historical data as shown in section 4. As can be seen in the graph, the sectoral development is again not replicating the historical development in the baseline specification where $\theta = 1$

Figure 15: Economic development with endogenous productivity growth ($\theta = 1$)



but once we adopt the specification with $\theta = 3$ the model replicates the stylized facts again. Further, the same analysis for parameter values as in the previous section has been repeated with very similar findings. The effect of variations for the new parameter η in the interval $[0, 0.05]$ and $\theta = 3$ can be seen in figure 17.

As one can see, the variations do not effect the general pattern of the development. The only differences are a faster transition when η is larger with minor differences in the share levels of the three sectors.

Possibile extensions of this paper could comprise the endogenization of technological change even more by integrating the lab-equipment model (Rivera-Batiz and Romer, 1991) into the secondary sector of the NP model in a broader formulation (Jones, 1999). Then the amount of investment in research and development will be determined by optimization. In the following section we explore the ability of one further modification to bring the model closer to the facts, namely the introduction of human capital accumulation.

Figure 16: Economic development with endogenous productivity growth ($\theta = 3$)

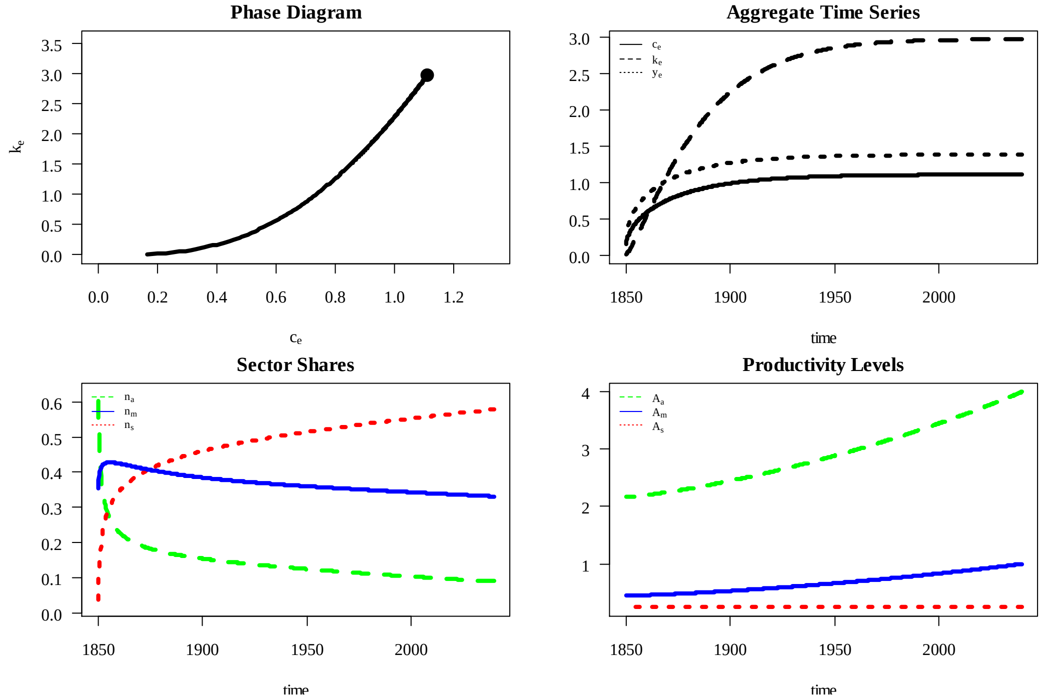
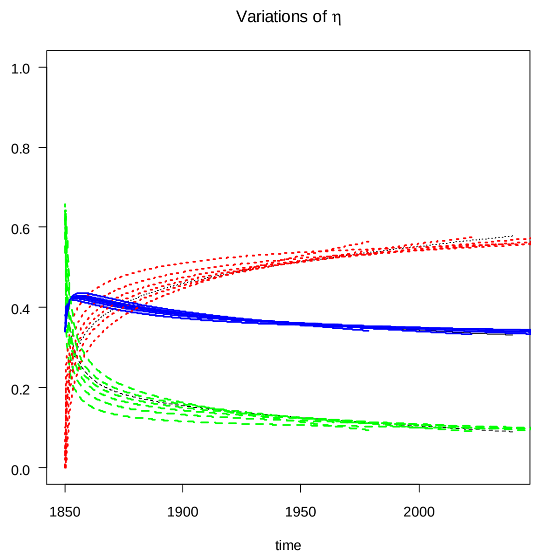


Figure 17: Parameter variations of R&D investment share η

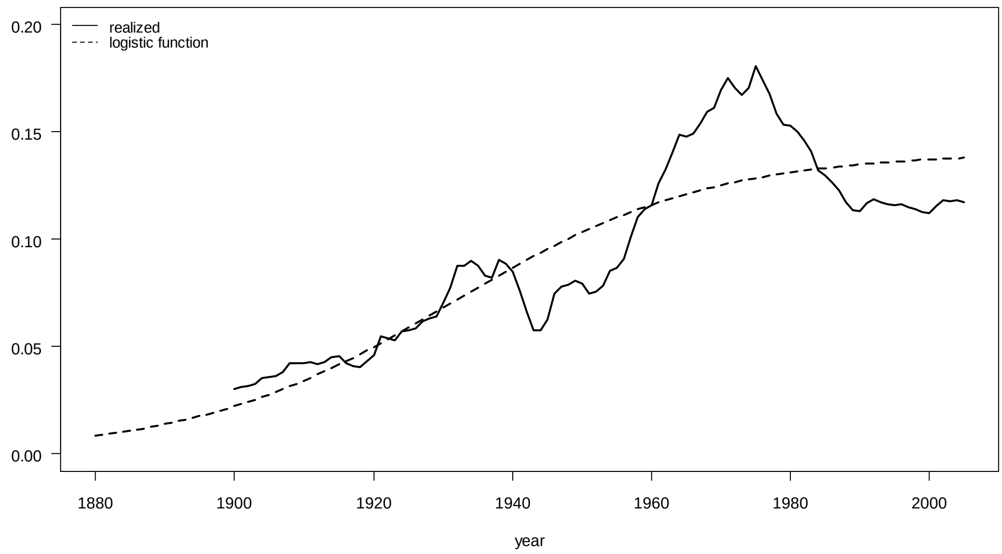


9 Human capital accumulation

One deviation of the model solution from the empirical facts that can not be resolved satisfactorily by the modifications introduced so far is that the employment share of the secondary sector (in which capital is accumulated) is bounded from below by the savings rate $1–c/y$ (see equation (14) of NP). One idea to lower this bound is to introduce human capital accumulation into the model. The idea is in the spirit of Lucas (1988) and assumes that in the secondary sector (and also in the tertiary sector) a certain fraction of the time that could be devoted to producing goods is actually used for education and training which leads to the accumulation of human capital. Thus, part of the time which is recorded now as production employment in the secondary sector is actually spent for human capital accumulation and not for the production of goods and services. The accumulation of human capital naturally takes place in the tertiary sector.

In the current production function labor input incorporates human capital which share is increasing in the course of time. As Witt (1997) points out, human labor can provide either a certain amount of physical work or a certain amount of mental work per unit of time. Here we argue that the amount of human capital which is needed differs dependending on the kind of work that shall be provided. Witt (1997) describes the evolution of human beings' tasks in the production process since the neolithic. He argues that up to the neolithic physical work was the predominant feature. With the domestication of animals and the usage of their physical work possibilities the production could be increased. With the beginning of the industrial revolution innovations were made that allowed using other kinds of energy (e.g., steam and oil) and human beings were able to become independent of human muscle power in the production process to a large extent. This allowed to increase the fraction of time devoting to mental work on the one hand but also made it necessary to learn how to use these alternatives of energy transference. The latter which Witt (1997) calls non-creative mental work first replaced physical work and this required knowledge could be transmitted via personal communication and imitation in the production process at least at the beginning. To be able to further increase the usage of non-human energy creative forms of mental work becomes more important and also non-creative mental work needed more human capital since machinery becomes more complicated and explicit technological knowledge had to be learned. In line with this argument, we argue that the stock of human capital increases in the course of economic development and therefore the fraction of time devoted to accumulating human capital also increases. Figure 18 shows the development of the time share devoted to human capital accumulation in the form of formal education in the U.S. from 1900 to 2005 as the solid line. The time share spent for human capital accumulation is computed as average weekly hours devoted to schooling of all people older than 14 years divided by the sum of average weekly hours devoted to school and average weekly hours worked by all people older than 14 years in a given year. As it can be seen in the figure the time fraction spent for human capital accumulation is very low (about three percent) around 1900 and rather steadily increases to a value of about 12 percent in 2005. An exceptional high ratio of education time to working time is the period around 1975 which is mainly due to a large economic recession at this time with high unemployment rates.[11] In sum, the long-run development of the fraction of education time can be explained best by a logistic function. The dashed line in figure 18 represents the fitted values of a non-linear least squares estimation.

Figure 18: Human capital investment of the U.S. since 1900



Data source: Ramey and Francis (2009), tables 2 and 3.

To introduce this rough idea into the model additional assumptions have to be made. First, it is assumed that the fraction of time spent on human capital accumulation is different in the three sectors since the tasks of labor in the production process are rather different. For simplicity it is assumed that the changes in the requirements for labor of the primary sector are of minor importance such that the effects are neglected in the subsequent analysis. Secondly, the overall time share spent on schooling κ is proportionally distributed in the secondary and the tertiary sector on average which is denoted by u. As before there is no unemployment so that the whole workforce belongs to one of the three main sectors and hence Eq. (7) applies. The time fraction which is devoted to produce goods and services exclusive of human capital production is then split according to

$$n_a + n_m(1-u) + n_s(1-u) = 1 - \kappa. \tag{33}$$

[11]Time-series to the unemployment rate in the U.S. can be found in: Bureau of Labor Statistics (BLS) – Household data annual averages: Table 1 Employment status of the civilian noninstitutional population, 1940 to date. [on the web: ftp://ftp.bls.gov/pub/special.requests/lf/aat1.txt].

The time fraction spent for the accumulation of human capital is proportionally distributed to the employment force of the secondary and the tertiary sector

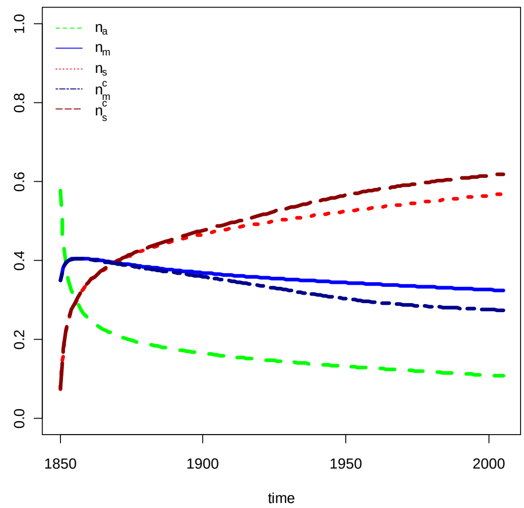
$$n_m u + n_s u = \kappa. \tag{34}$$

Since it is assumed that the human capital accumulation takes place in the tertiary sector, the corrected versions for the secondary and the tertiary sector shares are

$$n_m^c = n_m - \frac{\kappa}{n_m + n_s} n_m \text{ and } n_s^c = n_s + \frac{\kappa}{n_m + n_s} n_m. \tag{35}$$

In figure 19 the sectoral development of the model specification with non-homothetic preferences and asymptotic converging productivity growth rates of section 7 is plotted for the initial version (green, blue, and red lines) and corrected version (dark blue and dark red lines) of the sector shares.

Figure 19: Sectoral development including human capital



Of course, the assumptions only give a rough idea of the development when human capital accumulation is considered in the analysis. As can be seen in figure 19, the development of the primary sector does not change at all by assumption. The development of the tertiary sector is close to the original version but its share is increasing stronger as human capital accumulation becomes more important at the expense of the secondary sector. Of course, this kind of introducing human capital is very simple and the issue is treated in a very stylized way. However, the explicit consideration of human capital in the model potentially brings the model closer to the empirical evidence of the sectoral development.

Evidentely, this kind of taking account for human capital has the severe drawback that the effect of human capital on the production process itself is not captured. It would therefore be beneficial to consider these effects in a multi sector model version of Lucas (1988) where these effects could be investigated directly.

10 Conclusion

The basic insight of the numerical explorations reported so far is that the ability of the original NP model to explain the characteristic pattern of structural change among the three sectors of the private economy is limited. Major deficiencies are that the model is not able to account for the hump-shaped trajectory of the secondary sector. Our robustness analysis shows that such a hump-shaped pattern can be generated for the primary sector, but not for the secondary sector. This deficiency is probably linked to the counterfactually large initial share of the secondary sector which itself is due to the savings rate which effectively is a lower bound for this sector share in the model. Accordingly, while the initial share of the tertiary sector is reasonable, the initial share of the primary sector is unreasonably small.

The introduction of converging TFP growth rates between the three main sectors combined with the introduction of non-homothetic preferences yields essential improvements of the model. Non-homothetic preferences add a second driving force of structural change from the demand side. This intensifies structural change and also alters the direction of structural change. Now the model does not only replicate the monotonic decline of the primary sector share and the monotonic increase of the tertiary sector share, but most notably also the hump-shaped development of the secondary sector share, even though only in a very stylized version. The model still has the shortcoming that the secondary sector share is very large at the end of development caused by the fact that only the secondary sector's output can be invested. Therefore, the introduction of human capital accumulation is a reasonable way to lower the share of the secondary and to increase the share of the tertiary sector.

With respect to the criticism of neoclassical growth theory, the introduction of a simple version of endogenous technological change does not change the ability of the model to show the stylized trajectories when the converging TFP growth rates combined with non-homothetic preferences are applied.

What are the avenues of future research on economic growth and structural change? A very promising, although very demanding, undertaking would be the integration of structural change in unified growth theory. Starting point could be the model of Galor and Weil (2000). Combined with numerical analyses, freed from the straitjacket of analytical solutions, this could be a contribution to further enrich the explanatory power of this model class. From the perspective of empirical analysis, estimation of the parameters could be a complement to the calibration undertake in this paper (and many other papers). Moment matching methods like indirect inference (Gouriéroux et al., 1993; Gallant and Tauchen, 1996) could be used to match the trajectories from the numerical solution (which are conditional on the unknown parameters) to the empirical counterparts. The parameter estimates would be structural and could be directly compared to the calibrated values and judged regarding plausibility.

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